

The 1st Annual Vojtěch Jarník
International Mathematical Competition
Ostrava, 10th April 1991
Category I

Problem 1 Show that there is no arithmetic progression with three elements in the infinite geometric sequence $\{2^k\}_{k=0}^{\infty}$.

Problem 2 Compute the determinant

$$\det \begin{pmatrix} 0 & a_{12} & a_{13} & \dots & a_{1n} \\ -a_{12} & 0 & a_{23} & \dots & a_{2n} \\ -a_{13} & -a_{23} & 0 & \dots & a_{3n} \\ \dots & & & & \\ -a_{1n} & -a_{2n} & -a_{3n} & \dots & 0 \end{pmatrix},$$

where n is an odd number.

Problem 3 Let $[x]$ be the integer part of x . Find the limit

$$\lim_{n \rightarrow \infty} ((\sqrt{3} + 1)^n - [(\sqrt{3} + 1)^n]).$$

Problem 4 Let $f(x)$ be an even, twice continuously differentiable function and $f''(0) \neq 0$. Prove that there is an extremum of $f(x)$ at the point $x = 0$.